Lecture six

Topics that must be covered in this lecture:

- ***Finite automata with output***
- ***Moore Machine***
- ***Mealy Machine***
- ***Equivalence between Moore and Mealy Machines***

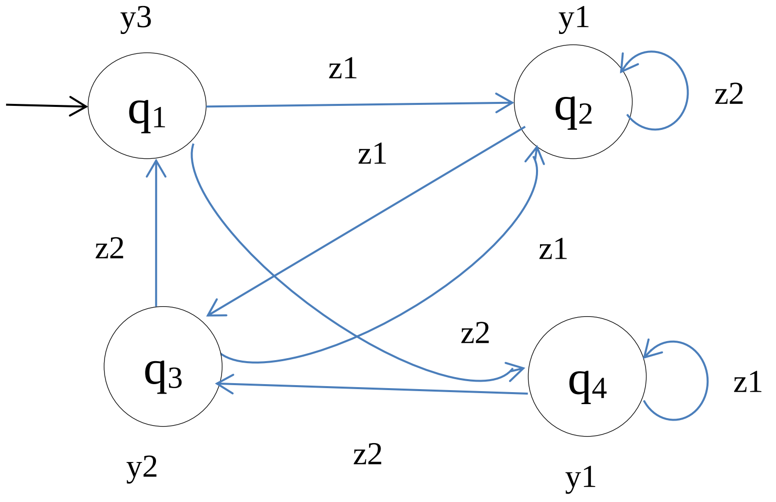
Finite automata with output

One limitation of the finite automata as we have defined it is that its output limited to a binary signal: " accept"/"reject". Models in which the output chosen from other alphabet have been considered. There are two distinct approaches; the output may be associated with the state (called a Moore machine) or with the transition (called a Mealy machine).

Moore Machine

A Moore machine is a six- tuple (Q, Σ, Δ, δ, λ, q0=start state), where Q, Σ, δ, q0 are as in the DFA. Δ is the output alphabet and λ is the mapping from Q to Δ giving the output associated with each state. The output of M in response to input a1a2…an, n≥0, is λ(q0) λ(q1)… λ(qn), where q0, q1, …qn is the sequence of states such that $\delta(q_{i-1},a_i)=q_i$ for $1 \leq i \leq n$. Note that any Moore machine gives output λ(q0)in response to input ϵ.

Example: Give the formal description for the following Moore machine M1 pictured below:



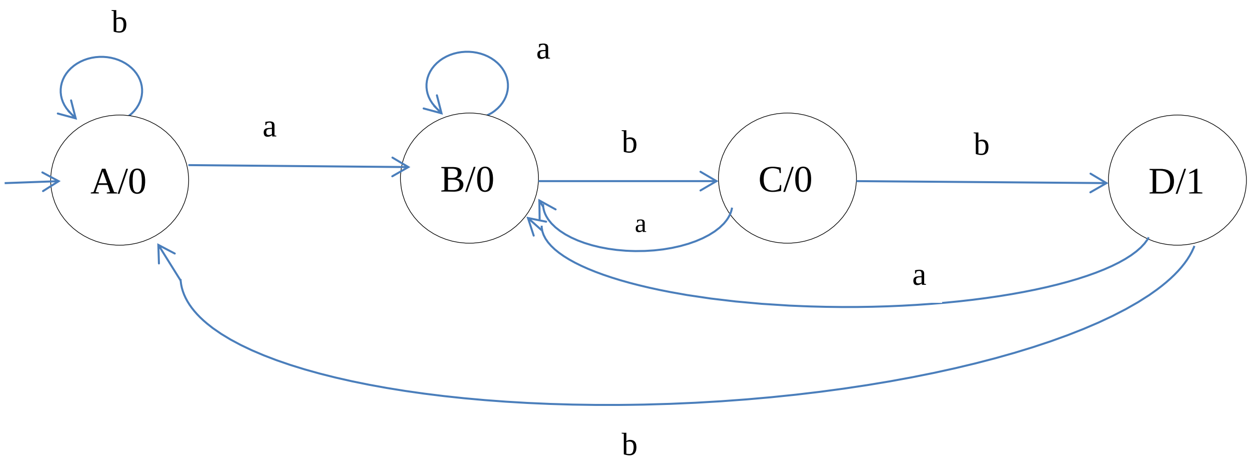
Sol:

M1=({q1,q2,q3,q4},{z1,z2},{y1,y2,y3},δ,λ,q1)

δ:Q× ∑→Q , λ:Q→ Δ

output state input	y3 q1	y1 q2	y2 q3	y1 q4
z1	q2	q3	q2	q4
z2	q4	q2	q1	q3

Example2: Give the formal description for the following Moore machine M2 pictured below, and give the output to the input string abbabb

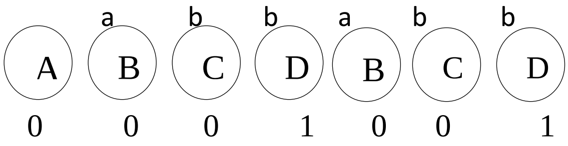


Sol:

- M2=({A,B,C,D},{a,b},{0,1},δ,λ,A)

output state input	0 A	0 B	0 C	1 D
a	B	B	B	B
b	A	C	D	A

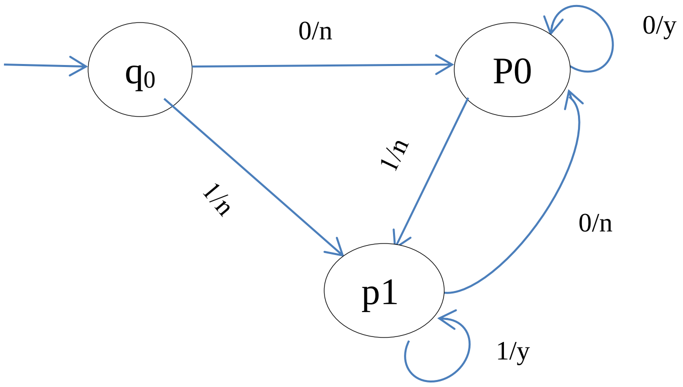
- OUTPUT:



Mealy Machine

A mealy machine is also a six – tuple (Q, Σ, Δ, δ, λ, q0=start state), where all is as in the Moore machine, except that λ maps Q$\times$ $\Sigma \rightarrow \Delta$. That is , λ(q,a) gives the output associated with the transition from state q on input a. the output of M in response to input a1a2…an is $\lambda(q_0,a_1)$, $\lambda(q_1,a_2)$,…, $\lambda(q_{n-1},a_n)$, where $q_0,q_1,\ldots,q_n$ is the sequence of states such that $\delta(q_{i-1},a_i)=q_i$ for $1\leq i \leq n$. Note that the sequence has length n rather than length n+1 as for the Moore machines and on input ϵ a Mealy machine gives output ϵ.

Example: Give the formal description for the following Mealy machine M3 pictured below:



Sol:

M3=({q0,p0,p1},{0,1},{n,y}, δ, λ,q0)

$\delta - table$

Q / Σ	q0	p0	p1
0	p0	p0	p0
1	p1	p1	p1

$\lambda - table$

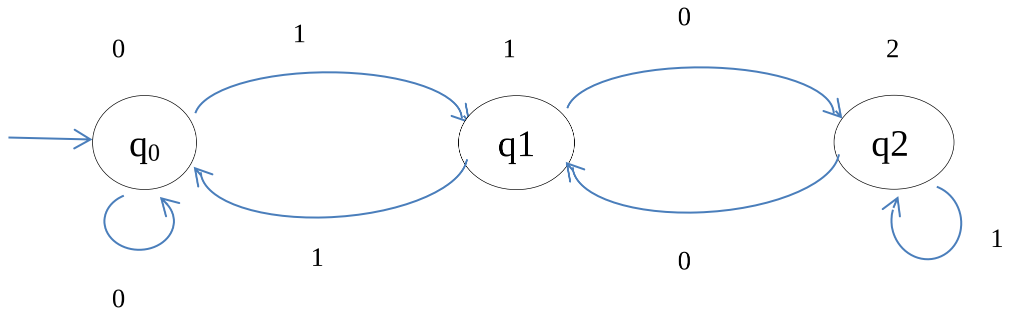
Q / Σ	q0	p0
0	n	y
1	n	n

Equivalence between Moore and Mealy Machines

Theorem: if M1= (Q, Σ, Δ, δ, λ, q0) is a Moore Machine, then there is a Mealy Machine M2 equivalent to M1.

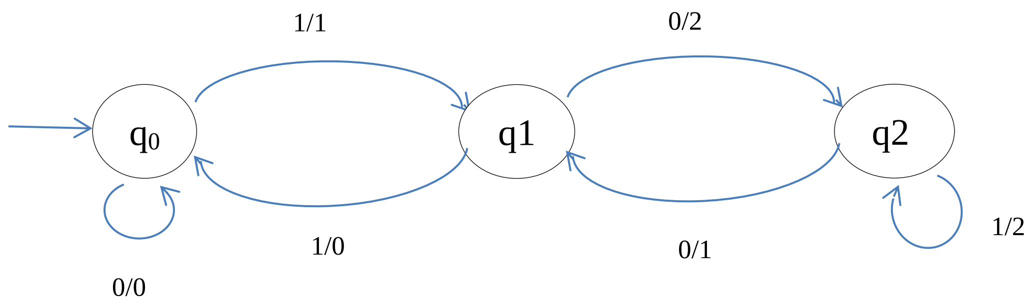
Proof: Let M2=(Q, Σ, Δ, δ, λ', q0) and define λ'(q,a) to be $\lambda(\delta$(q,a)) for all states q and input symbols a. then M1 and M2 enter the same sequence of states on the same input, and with each transition M2 emits the output that M1 associates with the state entered.

Example1: Construct an equivalence a Mealy Machine from a Moore Machine pictured below.



SOLUTION:

A mealy machine equivalence to a Moore machine :



Method of conversion :

λ'(q,a)= $\lambda(\delta$(q,a))

λ'(q0,0)= $\lambda(\delta$(q0,0))= λ (q0)=0

λ'(q0,1)= $\lambda(\delta$(q0,1))= λ (q1)=1

λ'(q1,0)= $\lambda(\delta$(q1,0))= λ (q2)=2

λ'(q1,1)= $\lambda(\delta$(q1,1))= λ (q0)=0

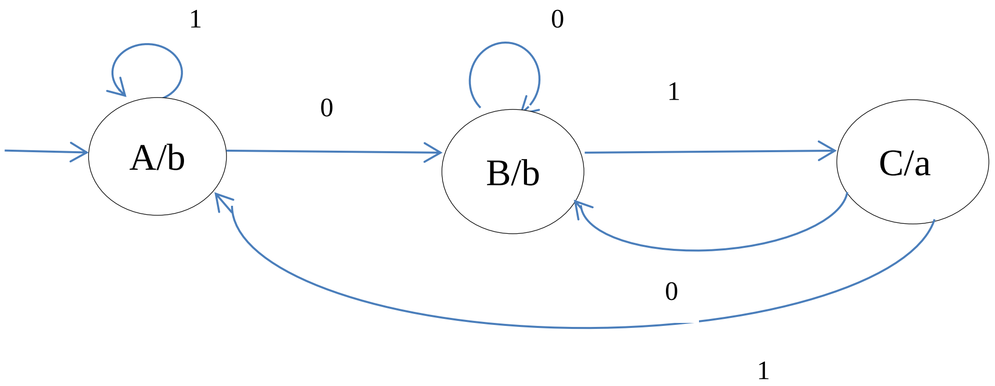
λ'(q2,0)= $\lambda(\delta$(q2,0))= λ (q1)=1

λ'(q2,1)= $\lambda(\delta$(q2,1))= λ (q2)=2

Transition table of a mealy machine:

Σ Q	**0**	**1**
q0	q0,0	q1,1
q1	q2,2	q0,0
q2	q1,1	q2,2

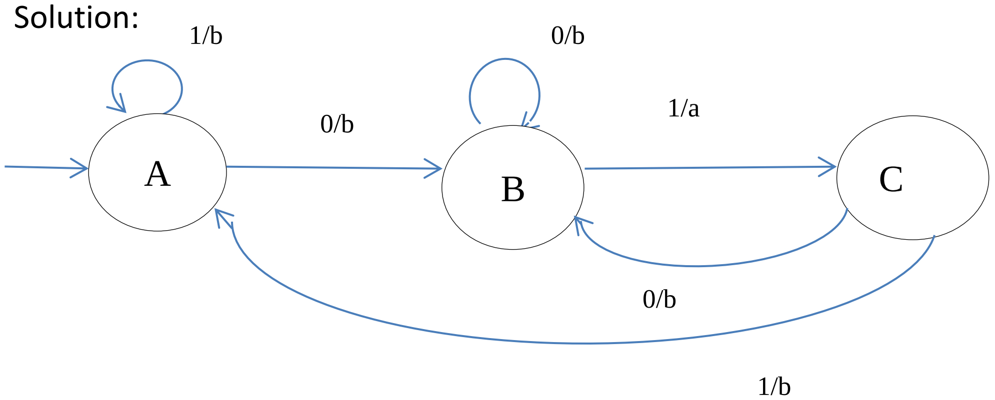
Example2: convert a Moore Machine to a Mealy Machine .



Transition table for a Moore machine:

Σ Q	**0**	**1**	**output**
A	B	A	b
B	B	C	b
C	B	A	a

Solution:



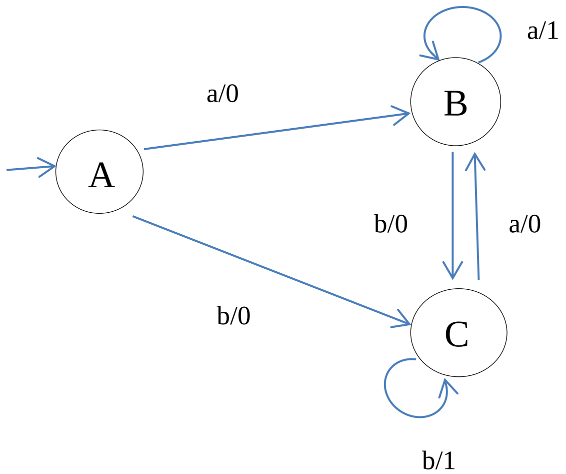
Transition table for a Mealy machine:

Σ / Q	**0**	**1**
A	B,b	A,b
B	B,b	C,a
C	B,b	A,b

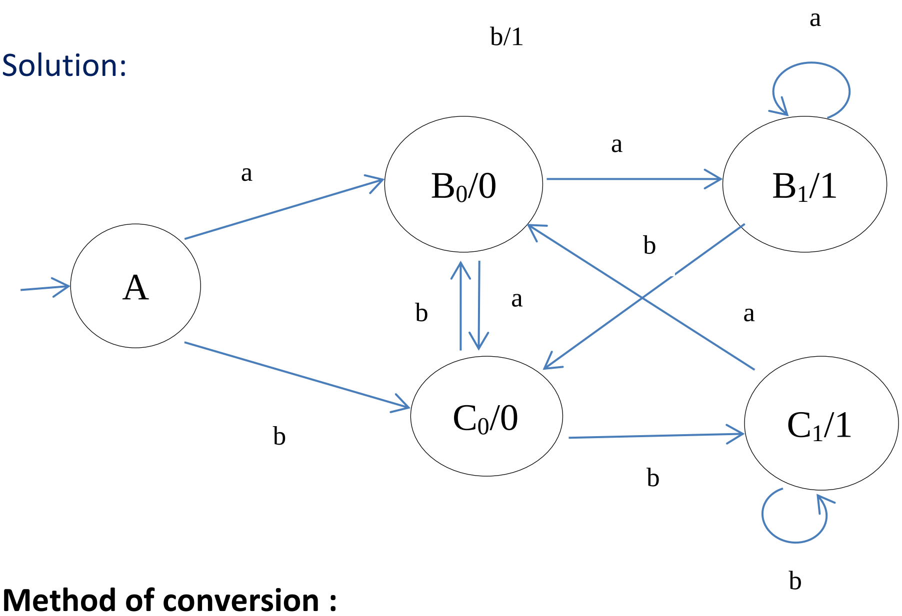
Theorem: let M1= (Q, Σ, Δ, δ, λ, q0) be a Mealy Machine, then there is a Moore machine M2 equivalent to M1.

Proof: Let M2=(Q $\times$ Δ, Σ, Δ, δ', λ', [q0,b0]) , where b0 is an arbitrary selected member of Δ. That is, the states of M2 are pairs [q,b] consisting of a state of M1 and an output symbol, in two examples below we will declare method of conversion .

Example1: convert the following Mealy machine to its equivalent Moore Machine.



Solution:



Method of conversion :

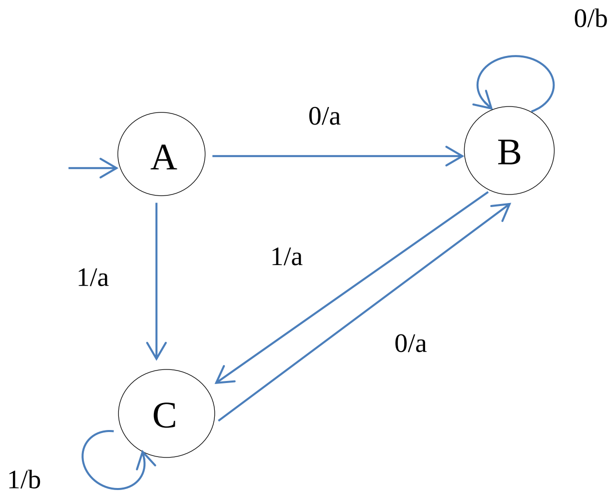
λ'([q,b],a)=[λ(δ(q,a), λ(q,a)]

λ'([q,b])=b

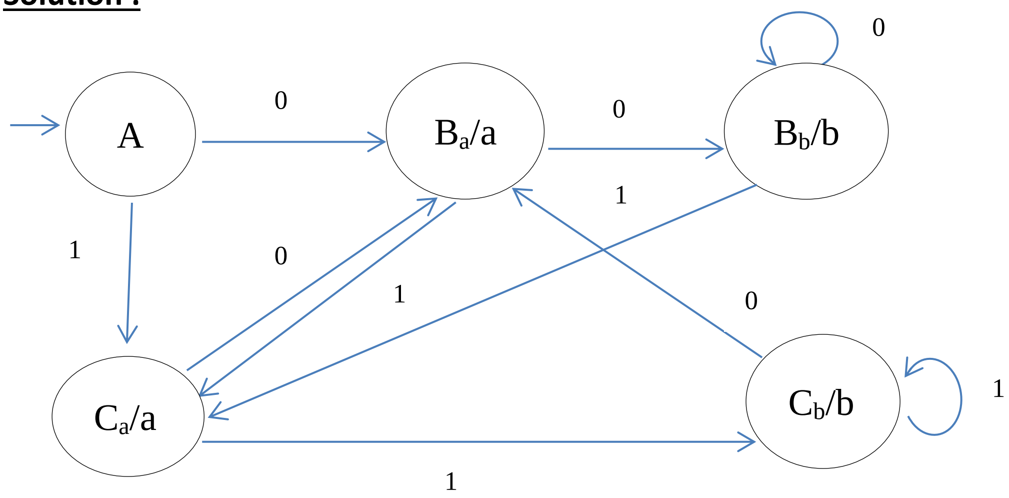
Δ={output symbol }

Note : we see every state have output except **A** does not have any output because the state A in original machine did not have any incoming edges.

Example2: convert the following Mealy machine to its equivalent Moore Machine.



<u>Solution :</u>



<u>Note :</u>

- When we convert a Moore machine to a Mealy machine →number of states were same .
- When we convert a Mealy machine to a Moore machine →number of states increased such that:

  In a Mealy to Moore: x and y →(x.y) no. of states at max

  States output