Find LCM

$$LCM (a, b) = |a * b| / GCD (a, b)$$

Example: Find LCM(4864,3458)

$$LCM (a, b) = |a * b| / GCD (a, b)$$

GCD (4864, 3458)

4864 = 3458 * 1 + 1406

3458= 1406 *2 + 646

1406 = 646 *2 + 114

646 = 114* 5 + 76

114 = 76 * 1 + 38

76 = 38 *2 + 0

GCD (4864, 3458) = 38

LCM (4864, 3458) = |3864 * 3458 | / GCD (4864, 3458)

= 16819712 / 38

= 442624

Modular Arithmetic

In mathematics, modular arithmetic is a system of arithmetic for integers.
Let a be an integer and m be a positive integer. We denote by a mod m the remainder when a is divided by m.

Examples:

9 mod 4 = 1

9 mod 3 =0

9 mod 10 =9: [(1) 9/10=0.9, (2) 0*10=0, (3) 9-0=9]

-13 mod 4 =3

-12 mod 7 = -5 mod 7 = 2 mod 7 =2, 9 mod7=2

Congruences

Let a and b be integers and m be a positive integer. We say that **a is congruent to b modulo m** if m divides a – b as m|(a-b)

We use the notation **$a \equiv b \pmod{m}$** to indicate that a is congruent to b modulo m.

In other words: $a \equiv b \pmod{m}$ if and only if

a mod m = b mod m.

Ex: 38≡ 14 (mod 12), because 38 − 14 = 24, which is a multiple of 12.

Another way to express this is to say that both 38 and 14 have the same remainder 2, when divided by 12

Ex: 29 ≡ 8 (mod 7)

Congruences

Examples:

Is it true that $46 \equiv 68 \pmod{11}$?
Yes, because $11 \mid (46 - 68)$.

Is it true that $68 \equiv 46 \pmod{22}$?
Yes, because $22 \mid (68 - 46)$.

For which integers z is it true that $z \equiv 12 \pmod{10}$?
It is true for any $z \in \{\ldots, -28, -18, -8, 2, 12, 22, 32, \ldots\}$

$-8 \equiv 7 \pmod{5}$

Modular Arithmetic Operations

1. [(a mod n) + (b mod n)] mod n = (a + b) mod n
2. [(a mod n) – (b mod n)] mod n = (a – b) mod n
3. [(a mod n) x (b mod n)] mod n = (a x b) mod n

e.g.

[(11 mod 8) + (15 mod 8)] mod 8 = 10 mod 8 = 2
(11 + 15) mod 8 = 26 mod 8 = 2

[(11 mod 8) – (15 mod 8)] mod 8 = –4 mod 8 = 4
(11 – 15) mod 8 = –4 mod 8 = 4

[(11 mod 8) x (15 mod 8)] mod 8 = 21 mod 8 = 5
(11 x 15) mod 8 = 165 mod 8 = 5

Representations of Integers

Let b be a positive integer greater than 1.
Then if n is a positive integer, it can be expressed **uniquely** in the form:

$$n = a_k b^k + a_{k-1} b^{k-1} + \dots + a_1 b + a_0,$$

where k is a nonnegative integer,
$a_0, a_1, \dots, a_k$ are nonnegative integers less than b,
and $a_k \neq 0$.

Example for b=10:

$$859 = 8 \cdot 10^2 + 5 \cdot 10^1 + 9 \cdot 10^0$$

Representations of Integers

Example for b=2 (binary expansion):

$(10110)_2 = 1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2^1 = (22)_{10}$

Example for b=16 (hexadecimal expansion):

(we use letters A to F to indicate numbers 10 to 15)

$(3A0F)_{16} = 3 \cdot 16^3 + 10 \cdot 16^2 + 15 \cdot 16^0 = (14863)_{10}$

Representations of Integers

How can we construct the base b expansion of an integer n?

First, divide n by b to obtain a quotient q_0 and remainder a_0, that is,

$n = bq_0 + a_0$, where $0 \leq a_0 < b$.

The remainder a_0 is the rightmost digit in the base b expansion of n.

Next, divide q_0 by b to obtain:

$q_0 = bq_1 + a_1$, where $0 \leq a_1 < b$.

a_1 is the second digit from the right in the base b expansion of n. Continue this process until you obtain a quotient equal to zero.

Representations of Integers

Example:
What is the base 8 expansion of $(12345)_{10}$?

First, divide 12345 by 8:
$12345 = 8 \cdot 1543 + 1$

$1543 = 8 \cdot 192 + 7$
$192 = 8 \cdot 24 + 0$
$24 = 8 \cdot 3 + 0$
$3 = 8 \cdot 0 + 3$

The result is: $(12345)_{10} = (30071)_8$.

Addition of Integers

Let $a = (a_{n-1}a_{n-2}...a_1a_0)_2$, $b = (b_{n-1}b_{n-2}...b_1b_0)_2$.

How can we add these two binary numbers?

First, add their rightmost bits:

$a_0 + b_0 = c_0 \cdot 2 + s_0$,

where s_0 is the **rightmost bit** in the binary expansion of $a + b$, and c_0 is the **carry**.

Then, add the next pair of bits and the carry:

$a_1 + b_1 + c_0 = c_1 \cdot 2 + s_1$,

where s_1 is the **next bit** in the binary expansion of $a + b$, and c_1 is the carry.

Addition of Integers

Continue this process until you obtain c_{n-1}.

The leading bit of the sum is $s_n = c_{n-1}$.

The result is:

$a + b = (s_n s_{n-1} \dots s_1 s_0)_2$

Addition of Integers

Example:
Add $a = (1110)_2$ and $b = (1011)_2$.

$a_0 + b_0 = 0 + 1 = 0 \cdot 2 + 1$, so that $c_0 = 0$ and $s_0 = 1$.

$a_1 + b_1 + c_0 = 1 + 1 + 0 = 1 \cdot 2 + 0$, so $c_1 = 1$ and $s_1 = 0$.

$a_2 + b_2 + c_1 = 1 + 0 + 1 = 1 \cdot 2 + 0$, so $c_2 = 1$ and $s_2 = 0$.

$a_3 + b_3 + c_2 = 1 + 1 + 1 = 1 \cdot 2 + 1$, so $c_3 = 1$ and $s_3 = 1$.

$s_4 = c_3 = 1$.

Therefore, $s = a + b = (11001)_2$.

Addition of Integers

How do we (humans) add two integers?

Example:

$$\begin{array}{r} 111 \\ 7583 \\ +\ 4932 \\ \hline 12515 \end{array}$$

carry

Binary expansions:

$$\begin{array}{r} 1\ \ 1 \\ (1011)_2 \\ +\ (1010)_2 \\ \hline (10101)_2 \end{array}$$

carry

Example: Write the number 37 as a base k=2

Solution: $37 = 2(18) + 1 \quad ; \quad q_1 = 18 > k.$

$$18 = 2(9) + 0 \quad ; \quad q_2 = 9 > k.$$

$$9 = 2(4) + 1 \quad ; \quad q_3 = 4 > k.$$

$$4 = 2(2) + 0 \quad ; \quad q_4 = 2 \geq k.$$

$$2 = 2(1) + 0 \quad ; \quad q_5 = 1 < k.$$

Then we put $q_5 = a_5$ and will get;

$$37 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= (100101)_2.$$

Example: Write the number 61469 as a base k=16

Solution: $61469 = 16(3841) + 13$

$$3841 = 16(240) + 1$$

$$240 = 16(15) + 0$$

$$61469 = 15(16^3) + 0(16^2) + 1(16) + 13$$

$$= (15000113)_{16}$$